EPSRC Engineering and Physical Sciences Research Council

ORCHID

Latent Force Models

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Gaussian Process Inference with Physical Priors

Aim to combine:

- **Differential model** of physical diffusion processes.
- Vague models of (possibly cyclic) processes expressed as a **Gaussian process**.

Latent Force Models (LFM) are traditionally Gaussian process priors for the solutions to differential equations. LFMs are hard to construct, are inflexible, suffer numerical instability and are innately stationary.

To alleviate these problems, we represent the GP prior using state-based models which are integrated into traditional state-based representations of the physical processes. The combined model is more flexible, numerically stable and computationally more efficient than traditional LFMs.

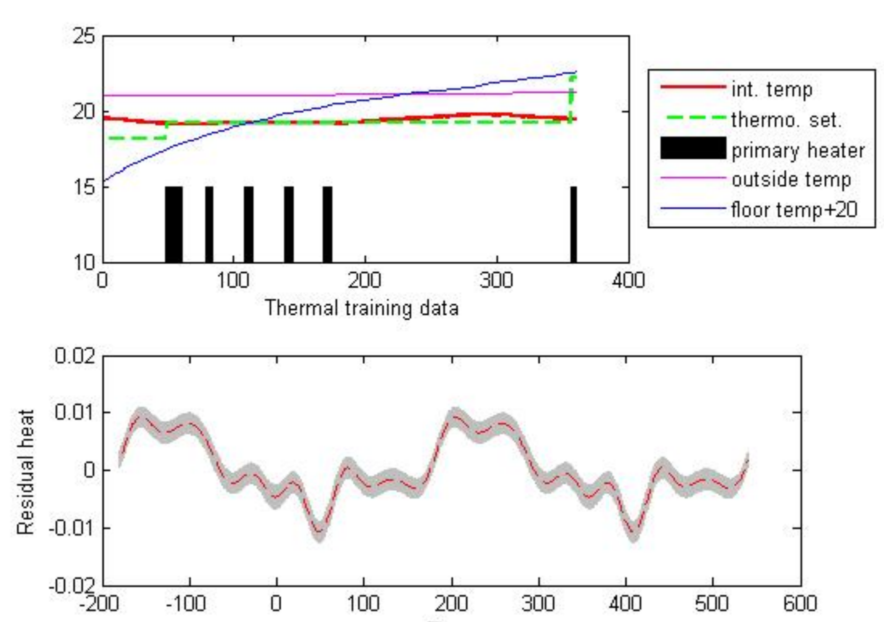
Temperature and Heating

Internal temperature (state variable) within a home depends on:

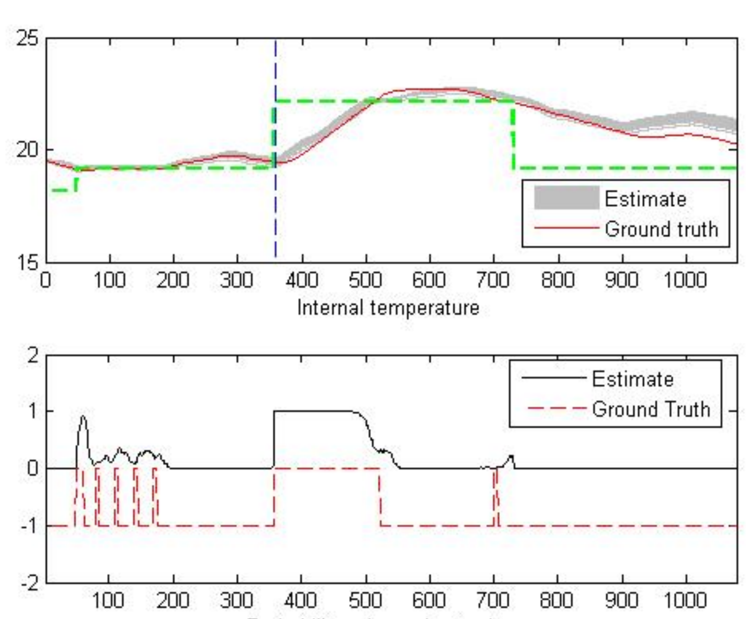
- Heater output
- Leakage of heat
- Residual unknown factors (e.g. occupancy)

The heat exchange process is expressed via differential equations and residual heat is expressed as a periodic GP.

Characterise thermal losses and model thermal processes within residential buildings.



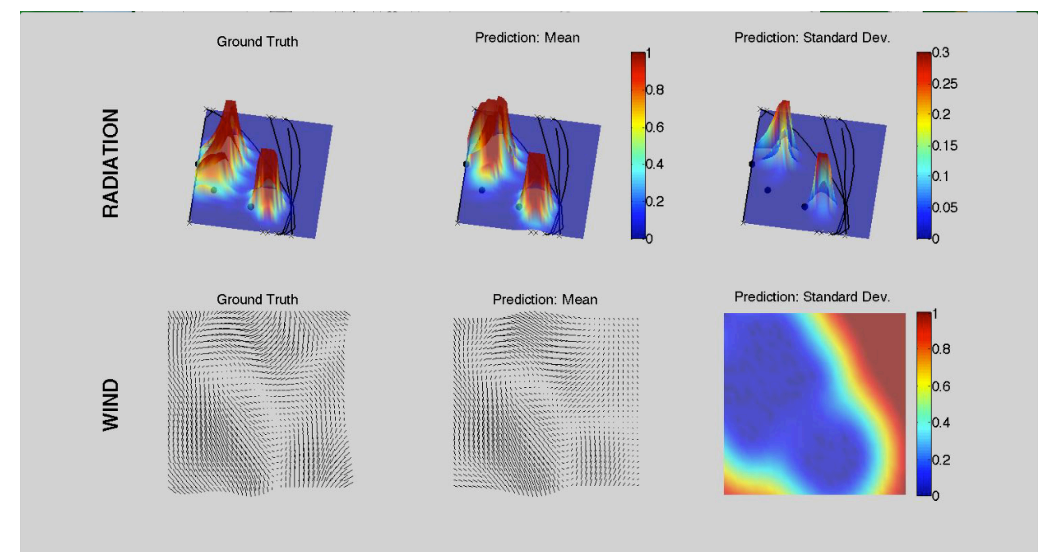
(a) Training data and inferred model.



(b) Predicted room temperature and primary heater output (pred mean= 161.7kw, truth=168kw)

Pollution Monitoring and Safe Route Planning

Aim to track pollution (or radiation cloud as per AtomicOrchid) using a differential model of cloud diffusion process and multi-output Gaussian process field model of wind speed and direction.



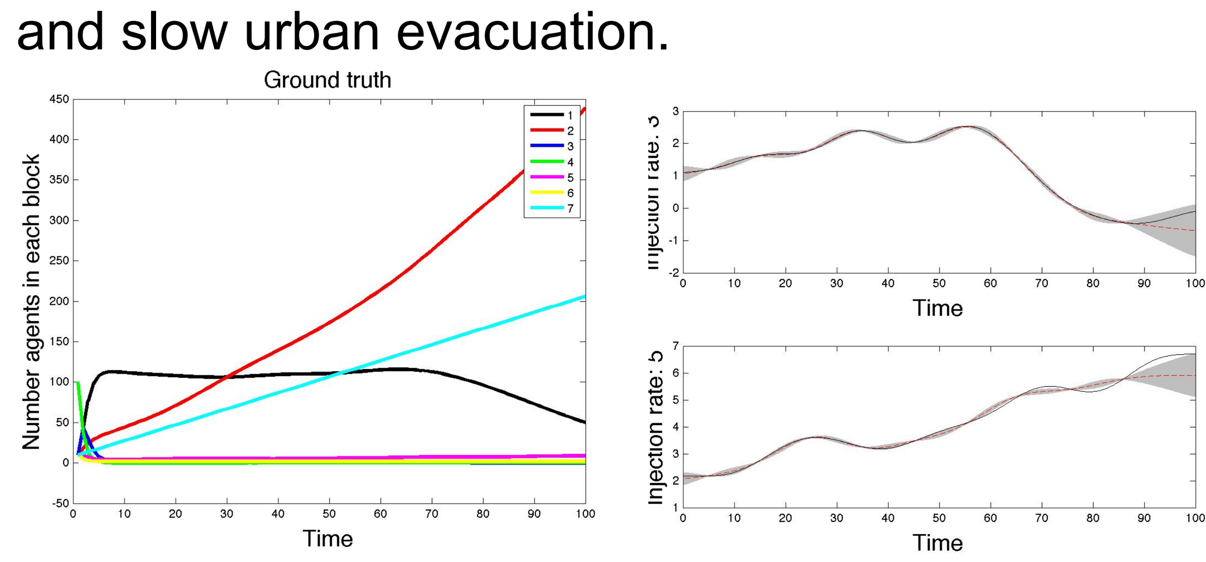
The wind is the latent force. Non-stationary (time evolving) wind dynamics are predicted by a Kalman filter by expressing the spatial GP within the KF process model and process noise covariance via the **KFGP** (Reece10).

Rescue service routes are chosen to avoid high radiation areas whilst still passing through waypoints such as hospitals. Paths are defined, again using a GP, and regions of high radiation density are avoided using waypoints which are reflections of the radiation field in the GP (S. Roberts).

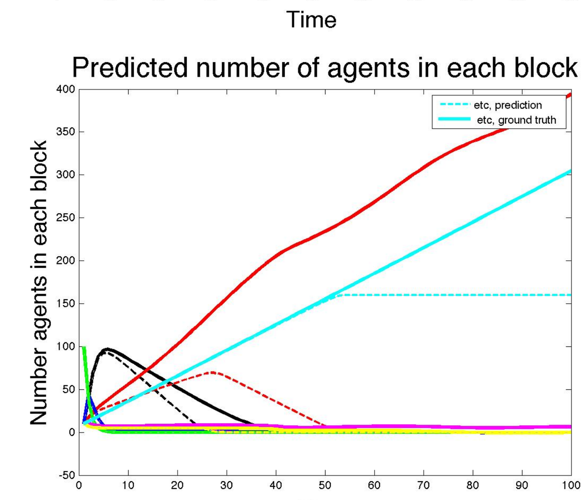
Urban Rescue

Aim to efficiently and rapidly evacuate urban area. Individuals stream out of buildings. Police officers direct at junctions where queues form. We model the process using differential equations for the M/M/1 queue and people from the buildings form the latent force. Firstly, learn building evacuation rates and then plan optimal queue service rates. Neglecting the building crowd can produce erroneous predictions and slow urban evacuation.

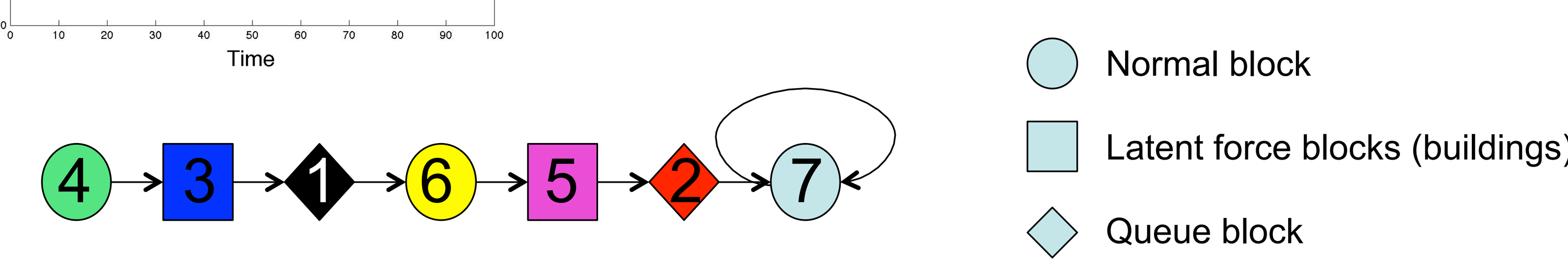
Screen shot from the Urban Rescue demonstrator.



(a) Service rates at blocks 1 (black) and 2 (red) are 2 per time unit during training.



(b) Ground truth and predicted block occupancy for service rates 5 and 3 at blocks 1 and 2, respectively. The dashed lines are predictions when no latent force model of building evacuation is used for both the training and prediction phases. In this case, the predicted block occupancy is severely under-estimated.



Optimal Control with Gaussian Process Disturbances

Consider **linear-quadratic-Gaussian** control with disturbance drawn from a stationary GP with (near) arbitrary covariance function, K. One option is to integrate the linear dynamic system and disturbance kernel into a system wide covariance function subject to the Riccati equation. This is difficult (and we argue impossible). Alternatively, convert the disturbance GP into a stochastic differential equation (Hartikainen10, Reece13) and integrate into traditional linear system equations.

The Riccati solution can be found using classical state-based methods. Note, in the solution, $u(t) = -L(t)\hat{x}(t)$, L is independent of the stochastic part of the disturbance. Extracting the deterministic part only would be difficult, if not impossible, when using the traditional LFM approach.

Problem is to find control, u(t), for stationary process, x(t),

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + v(t) \qquad v \sim \mathrm{GP}(0, K)$$

$v \sim \mathrm{GP}(0, K)$ → Markovianise →

$$W(t) = \begin{bmatrix} v(t) \\ \dot{v}(t) \\ \ddot{v}(t) \\ \vdots \end{bmatrix} \qquad J = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \qquad \dot{W}(t) = G(t)W(t) + J\omega(t)$$

$$\omega \sim N(0, \Omega) \text{ and } \omega \text{ is i.d.d.}$$

Thus,

$$\begin{bmatrix} \dot{x}(t) \\ \dot{W}(t) \end{bmatrix} = \begin{bmatrix} A(t) & 1\,0\cdots0 \\ 0 & G(t) \end{bmatrix} \begin{bmatrix} x(t) \\ W(t) \end{bmatrix} + \begin{bmatrix} B(t)u(t) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ J \end{bmatrix} \omega(t)$$

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